Int. J. Solids Structures Vol. 27, No. 4, pp. 521–522, 1991
Printed in Great Britain.
0020-7683/91 $3.00+ .00

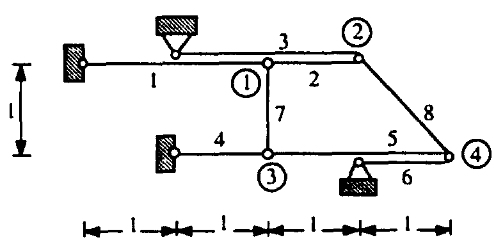
AUTHORS' CLOSURE

Professor Kuznetsov's most substantial criticism is contained in Section 3. He states that the assembly shown below in Fig. 6a is a counter-example to our work, since a sign-definite combination of quadratic forms does not exist, and yet the assembly is (according to him) a first-order infinitesimal mechanism.

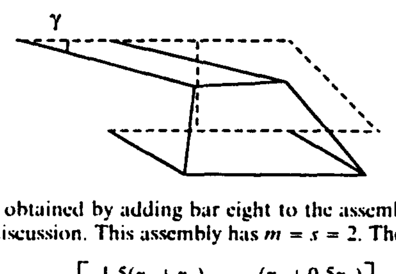
We have analyzed the assembly of Fig. 6a in detail. We agree that a sign-definite combination of quadratic forms does not exist. However, we disagree with Kuznetsov's claim that the asssembly is a first-order infinitesimal mechanism, since it can be shown that the mechanism indicated in Fig. 6b involves third-order elongations of bar three, all other bars being completely rigid. Therefore, the assembly of Fig. 6a is a *second-order infinitesimal mechanism*. This result is perfectly consisent with the non-existence of a positive definite quadratic form. It appears that Kuznetsov's examination of the assembly, on which he evidently based his conclusion, was not sufficiently thorough to detect this particular mechanism.

Thus the assembly of Fig. 6a, far from being a destructive counter-example to our analysis, is actually in complete accord with it. In fact, it is a nice example of a non-trivial "finite" mechanism with $m = s = 2$; see our comments at the end of Section 5.

The remainder of Kuznetsov's lengthy remarks may be summarized as follows. For several years we have been making progress in setting up matrix methods for analyzing general assemblies of rods and joints, and in particular for classifying the order of any infinitesimal mechanisms which may exist in them. Kuznetsov claims that virtually nothing in our work is an advance on older methods which involve, essentially, the construction of quadratic forms in a way which he describes as being "very direct and simple". We believe



a)



b)

Fig. 6. (a) Plane assembly obtained by adding bar eight to the assembly of Fig. 5, as described in Section 3 of Kuznetsov's discussion. This assembly has $m = s = 2$. The matrix Q is

$$Q = \begin{bmatrix} 1.5(\alpha_1+\alpha_2) & -(\alpha_1+0.5\alpha_2) \\ -(\alpha_1+0.5\alpha_2) & 0.5(\alpha_1-\alpha_2) \end{bmatrix},$$

and is *sign-indefinite*. This implies that the assembly is *not a first-order infinitesimal mechanism*. (b) If bar one rotates by γ, and all bars except three are inextensional, then in the mechanism shown bar three undergoes the elongation $-\gamma^3+0(\gamma^2)$ (in the drawing, bar three shortens by approximately 1%). Therefore, the assembly shown is a *second-order infinitesimal mechanism*. We found other, rather similar, mechanisms which involve third-order elongations of another bar while all other bars are inextensional.

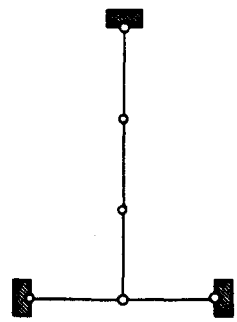


Fig. 7. Plane assembly with $m = 2$ and $s = 1$, which is incorrectly shown to be a second-order infinitesimal mechanism by Kuznetsov (1988). Tarnai (1990) has shown that this assembly is a third-order infinitesimal mechanism.

that we have already answered most of these points in Section 5; but here we would like to make two remarks.

First, Kuznetsov states in Section 2, that "the quadratic form in all variables is obtained instantly, *without any calculation*, as a linear combination of constraint functions weighted by their respective tension coefficients". This is a misleading statement since, in general, considerable prior computational effort is required to obtain not only the "independent displacements" (i.e. what we describe as the *inextensional mechanisms*) but also the "tension coefficients" (i.e. what we describe as the *states of self-stress*). In our view, a matrix formulation is—in the present age of inexpensive computation—the most obvious and economical way of doing the calculations.

Second, we remark that in spite of the high-flown claims which Kuznetsov makes, the examples which he uses to illustrate his methods in his own papers are generally much simpler than ours. Indeed, in the particular case of the assembly shown above in Fig. 7, Kuznetsov (1988) is obliged to resort to an *ad hoc* reduction into two sub-assemblies in order to obtain results. In this context, an important consideration, of which many authors appear to be unaware, is that the "invitingly simple" form of the constraint equations used by, e.g. Kuznetsov (1988) is not suitable for a general analysis of higher-order mechanisms. This is because the constraint equation [with the symbols of our formula (10)],

$$(x_q - x_r)^2 + (y_q - y_r)^2 + (z_q - z_r)^2 - l_p^2 = 0$$

is only equivalent to the true constraint equation

$$\sqrt{(x_q - x_r)^2 + (y_q - y_r)^2 + (z_q - z_r)^2} = l_p$$

up to the second-order when the square root is expanded as a Taylor Series. This point has been addressed, and resolved for assemblies with $m = s = 1$, by Pellegrino (1986).

We can but admire Professor Kuznetsov's tenacity in repeatedly reiterating his views on these matters both in this journal and also in the *Journal of Applied Mechanics*; but we are sorry that he feels obliged to lard his comments with so much gratuitous abuse.

C. R. Calladine and S. Pellegrino
Cambridge University Engineering Department
Trumpington Street
Cambridge CB2 1PZ, U.K.

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